BRILLIANT TUTORIALS®

IIT 2009
PT2/MPC/P(I)/SOLNS

B.MAT PART TEST 2

FOR OUR STUDENTS

TOWARDS

IIT-JOINT ENTRANCE EXAMINATION, 2009

PAPER I – SOLUTIONS
MATHEMATICS – PHYSICS – CHEMISTRY

PART A : MATHEMATICS

SECTION I

1. (A) $\sin\left(\frac{\pi}{20}\right) + \cos\left(\frac{\pi}{20}\right) = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{20}\right) = \sqrt{2}\sin\left(\frac{3\pi}{10}\right)$

$\sin\left(\frac{3\pi}{10}\right) > \sin\left(\frac{\pi}{4}\right) \Rightarrow \sqrt{2}\sin\left(\frac{3\pi}{10}\right) > 1$

Also $\sqrt{2}\sin\left(\frac{3\pi}{10}\right) < \sqrt{2}$

$\therefore \left[\sin\left(\frac{\pi}{20}\right) + \cos\left(\frac{\pi}{20}\right)\right] = 1$

$\cos\left(\frac{\pi x}{2}\right) = 1$

$\Rightarrow \frac{\pi x}{2} = 2n\pi$

$\Rightarrow x = 4n$

2. (D) The equation is $(\sin^2 x - 1)^2 + (\cos^2 y - 1)^2 + 2(\sin x - \cos y)^2 = 0$

$\Rightarrow \sin^2 x = 1, \quad \cos^2 y = 1, \quad \sin x = \cos y$

$\sin x = 1, \quad \cos y = 1, \quad \sin x - \cos y = 0$

$\sin x = -1, \quad \cos y = -1, \quad \sin x - \cos y = 0$

3. (C) Let $\frac{1}{2}\cos^{-1} x = \theta$

Then $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 1$

$$\Rightarrow \frac{2\,(1 + \tan^2\theta)}{1 - \tan^2\theta} = 1$$

$$\Rightarrow \frac{2}{\cos 2\theta} = 1$$

$$\Rightarrow \frac{2}{x} = 1 \Rightarrow x = 2$$

But $\cos^{-1} x$ is not defined for x = 2

4. (B) $\frac{b^2}{c^2} = \frac{\sin^2\left(\frac{4\pi}{9}\right)}{\sin^2\left(\frac{2\pi}{9}\right)}$

$$\Rightarrow \frac{b^2 - c^2}{c^2} = \frac{\sin\left(\frac{6\pi}{9}\right)\sin\left(\frac{2\pi}{9}\right)}{\sin^2\left(\frac{2\pi}{9}\right)}$$

$$= \frac{\sin A}{\sin C} = \frac{a}{c}$$

$b^2 - c^2 = ac$

5. (A) The three points lie on $x^2 + y^2 = 13^2$

$\therefore$ circumcentre S is the origin.

Centroid is $\left(\frac{13(\sin\theta + \cos\theta) + 5}{3}, \frac{13(\sin\theta - \cos\theta) + 12}{3}\right)$

If H (α, β) is the orthocentre

then, $\frac{\alpha}{3} = \frac{13(\sin\theta + \cos\theta) + 5}{3}$, $\frac{\beta}{3} = \frac{13(\sin\theta - \cos\theta) + 12}{3}$

[∵ SG : GH = 1 : 2]

$\sin\theta + \cos\theta = \frac{\alpha - 5}{13}$; $\sin\theta - \cos\theta = \frac{\beta - 12}{13}$

$$\therefore \left(\frac{\alpha - 5}{13}\right)^2 + \left(\frac{\beta - 12}{13}\right)^2 = 2$$

Locus of (α, β) is $(x - 5)^2 + (y - 12)^2 = 338$

6. (C) A is (3, 0), B is (0, 2) [where AB is the diagonal]

Midpoint of AB $= \left(\frac{3}{2}, 1\right)$ and $\frac{1}{2}$ AB $= \frac{\sqrt{13}}{2}$

Slope of the other diagonal is $\frac{3}{2}$

$\therefore$ the equation of the other diagonal is

$$\frac{x - \frac{3}{2}}{\frac{2}{\sqrt{13}}} = \frac{y - 1}{\frac{3}{\sqrt{13}}} = \frac{\sqrt{13}}{2}$$

One extremity is $\left(\frac{5}{2}, \frac{5}{2}\right)$ and the other extremity is $\left(\frac{1}{2}, -\frac{1}{2}\right)$

$\therefore$ the required ratio is 3 : 2

SECTION II

7. (A), (C)

$\sin^2 \pi x - \sin^2 \pi y = \frac{1}{2}$

$\sin \pi(x + y) \sin \pi(x - y) = \frac{1}{2}$

$\sin \pi (x + y) = 1$

$\pi (x + y) = (4n + 1) \frac{\pi}{2}$

$x + y = 2n + \frac{1}{2}$

$x - y = \frac{1}{6}$

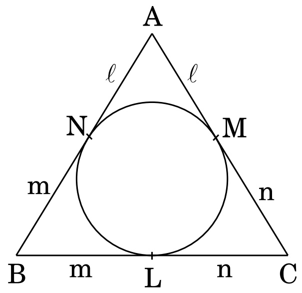
Solving $x = n + \frac{1}{3}$; $y = n + \frac{1}{6}$

(A), (C) are got by putting n = 0 and n = 1

(B), (D) are got by putting $n = \frac{1}{2}$ and $n = \frac{1}{3}$ which are not correct.

8. (A), (B), (C)

From the **Figure**



$2s = a + b + c$

$= (m + n) + (n + \ell) + (\ell + m)$

$\Rightarrow s = \ell + m + n$

$s - a = \ell + m + n - (m + n) = \ell$

$s - b = \ell + m + n - (n + \ell) = m$

$s - c = \ell + m + n - (\ell + m) = n$

$\Delta^2 = s(s - a)(s - b)(s - c) = (\ell + m + n)\,\ell mn$

$$r^2 = \frac{\Delta^2}{s^2} = \frac{(\ell + m + n)\,\ell mn}{(\ell + m + n)^2} = \frac{\ell mn}{\ell + m + n}$$

9. (C), (D)

The equation of the line is

$$\frac{x - 2}{\cos\alpha} = \frac{y - 4}{\sin\alpha} = r$$

Then $6 - 2 = PL\cos\alpha \Rightarrow PL = \frac{4}{\cos\alpha}$

$8 - 4 = PM\sin\alpha \Rightarrow PM = \frac{4}{\sin\alpha}$

$$PL + PM = 4\left[\frac{\cos\alpha + \sin\alpha}{\cos\alpha\sin\alpha}\right]$$

$$= \frac{8\sqrt{2}\sin\left(\frac{\pi}{4} + \alpha\right)}{\sin 2\alpha}$$

$$\left(\frac{4}{PL}\right)^2 + \left(\frac{4}{PM}\right)^2 = \cos^2\alpha + \sin^2\alpha = 1$$

10. (A), (C)

Let the vertex be (0, k).

The diagonals are parallel to the angle bisectors of the given lines.

i.e., parallel to $\frac{x - y + 2}{\sqrt{2}} = \pm\left(\frac{7x - y + 3}{\sqrt{50}}\right)$

One slope = 2 and another slope = $-\frac{1}{2}$

$\frac{k - 2}{-1} = 2$ or $\frac{k - 2}{-1} = \frac{-1}{2}$

$\Rightarrow$ k = 0 or $k = \frac{5}{2}$

SECTION III

11. (A) sin A cos A + sin B cos B + sin C cos C ≤ $\frac{1}{2}\left(\sin A + \sin B + \sin C\right)$

i.e., sin 2A + sin 2B + sin 2C ≤ sin A + sin B + sin C

i.e., 4 sin A sin B sin C ≤ 4 $\cos\left(\frac{A}{2}\right)\cos\left(\frac{B}{2}\right)\cos\left(\frac{C}{2}\right)$

i.e., $\sin\left(\frac{A}{2}\right)\sin\left(\frac{B}{2}\right)\sin\left(\frac{C}{2}\right) \le \frac{1}{8}$

12. (A) Let $\cos^{-1} x = y \Rightarrow x = \cos y$

$\therefore$ the expression is $\cos^{-1}\left[\frac{\cos y}{2} + \frac{1}{2}\sqrt{3 - 3\cos^2 y}\right]$

$$= \cos^{-1}\left[\frac{1}{2}\cos y + \frac{\sqrt{3}}{2}\sin y\right]$$

$$= \cos^{-1}\cos\left(\frac{\pi}{3} - y\right) = \frac{\pi}{3} - y$$

13. (A) sin x + 2 sin λ cos x = 2

$\Rightarrow \sqrt{1 + 4\sin^2\lambda} \ge 2$

$\sin^2\lambda \ge \frac{3}{4}$

14. (D) The distance between the 2 points is 10. Since the length of the perpendicular from (– 2, 3) is 9 which is < 10, a line can be drawn through (4, – 5).

SECTION IV

15. (B) 1st equation $2\sec^2\theta - 5\sec\theta + 2 = 0$

Roots are $\sec\theta = 2$;

$\sec\theta = \frac{1}{2}$ (not possible);

$\cos\theta = \frac{1}{2}$ ... (1)

$\sin\theta = -\frac{\sqrt{3}}{2}$ ... (2)

For (1) general solution is $\theta = 2n\pi \pm \frac{\pi}{3}$

For (2) general solution is $\theta = n\pi - (-1)^n \frac{\pi}{3}$

Common solution between 0 and 2π is $\frac{5\pi}{3}$

General solution is $2n\pi + \frac{5\pi}{3}$, $n \in Z$

16. (D) $\cos\theta = -\frac{1}{\sqrt{2}}$; $\tan\theta = 1$

$\theta = 2m\pi \pm \frac{5\pi}{4}$; $\theta = m\pi + \frac{\pi}{4}$

Common solution between 0 and 2π is $\frac{5\pi}{4}$

General solution is $2m\pi + \frac{5\pi}{4}$

$$= 2m\pi + \pi + \frac{\pi}{4}$$

$$= (2m + 1)\pi + \frac{\pi}{4}$$

17. (A) 2 cos x cos 6x = – 2

cos 7x + cos 5x = – 2

which is possible only when cos 7x = – 1, cos 5x = – 1

Common value of x is odd multiple of π

$x = (2n + 1)\pi,\ n \in I$

18. (C) Inradius = $\frac{4}{2}\cot\frac{\pi}{3} = \frac{2}{\sqrt{3}}$

It is the circumradius for the square.

Area of the square = $\dfrac{4\left(\frac{2}{\sqrt{3}}\right)^2}{2}\sin\left(\frac{2\pi}{4}\right) = \frac{8}{3}$

19. (B) Let r be the radius of the circle

$$a = 2r\tan\left(\frac{\pi}{3}\right) = 2r\sqrt{3}$$

$$b = 2r\tan\left(\frac{\pi}{4}\right) = 2r$$

$$c = 2r\tan\left(\frac{\pi}{6}\right) = \frac{2r}{\sqrt{3}}$$

$b^2 = ac$

20. (D) $\frac{r}{R} = \dfrac{\cot\left(\frac{\pi}{6}\right)}{\operatorname{cosec}\left(\frac{\pi}{6}\right)} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$$\frac{r^2}{R^2} = \frac{3}{4}$$

21. (A) $(\alpha\beta)(\beta\gamma)(\gamma\alpha) = 64$

$\alpha\beta\gamma = 8 \Rightarrow \gamma = 4; \quad \beta = 1; \ \alpha = 2$

$\Rightarrow 3a = 3 \Rightarrow a = 1$

$3b = 5 \Rightarrow b = \frac{5}{3}$

$3c = 6 \Rightarrow c = 2$

$\Rightarrow abc = \frac{10}{3}$

22. (A) Centroid $= \left[\frac{\alpha^2 + \beta^2 + \gamma^2}{3}, \frac{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}}{3}\right]$

$= \left[7, \frac{7}{12}\right]$

23. (C) P is $\left(4, \frac{1}{2}\right)$; Q is $(1, 1)$; R is $\left(16, \frac{1}{4}\right)$

PART B : PHYSICS

SECTION I

24. (A) Force applied, $F = m\frac{dv}{dt}$

Gain in kinetic energy, $\Delta E = \frac{1}{2}\ mv^2$

$$m\frac{dv}{dx} \times \frac{dx}{dt} = F$$

$$m\int v\ dv = \int F\ dx$$

$$\Delta E = \frac{1}{2}\ mv^2 = Fx$$

$$v = \frac{dx}{dt} = \left(\frac{2Fx}{m}\right)^{1/2}$$

$$\int \frac{dx}{x^{1/2}} = \left(\frac{2F}{m}\right)^{1/2} \int dt$$

$$2x^{1/2} = \left(\frac{2F}{m}\right)^{1/2} t$$

$$\Rightarrow x = \frac{Ft^2}{2m} \qquad \therefore x \propto t^2$$

25. (B) Initial K.E. $E_i = \frac{1}{2}\ mv_i^2$

Final K.E. = E_f

$$\frac{\left(E_i - E_f\right)}{E_i} = F$$

$$\Rightarrow E_f = (1 - F)E_i$$

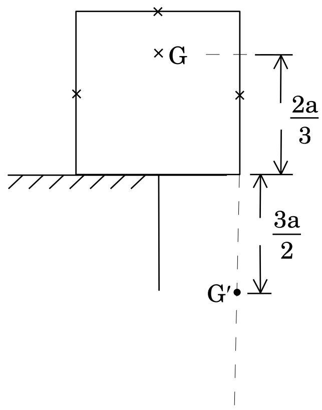
$$\frac{1}{2}\ mv_f^2 = (1 - F)\ \frac{1}{2}\ mv_i^2$$

Initial velocity $v_i = \sqrt{2gh}$

Final velocity $v_f = \sqrt{(1 - F)}\ v_i = \sqrt{(1 - F)\ 2gh}$

26. (B) Initially the centre of mass G is at a distance

$$\frac{\frac{m}{3}\left(\frac{a}{2}\right)+\frac{m}{3}(a)+\frac{m}{3}\,\frac{a}{2}}{m}=\frac{2a}{3}\text{ from the table.}$$



Change in position of centre of mass of the chain, $\Delta h=\left(\frac{2a}{3}+\frac{3a}{2}\right)=\frac{13a}{6}$

Work done by gravity

$$=mg(\Delta h)=mg\cdot\frac{13a}{6}$$

Gain in K.E. $=\frac{mv^2}{2}$

$$v=\sqrt{2\times\frac{13}{6}\,ga}=\sqrt{\frac{13}{3}\,ga}$$

27. (D) For the translational motion of the cylinder

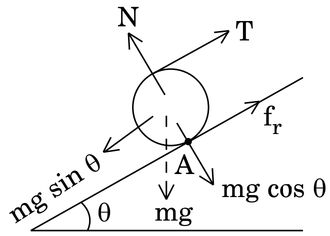
$f_r+T-mg\sin\theta=ma$

For the rotational motion of the cylinder about centre of mass

$T(R)-(f_r)R=I_{CM}\,\alpha$

To balance this way, $a=\alpha=0$

$$T=\frac{mg\sin\theta}{2}\qquad T=f_r$$



28. (B) $\frac{\Delta V}{V_0}=-\frac{\Delta P}{B}=\frac{-1.6\times10^{7}}{5\times10^{9}}=-0.32\times10^{-2}$

29. (D) In the case of a body that rolls without sliding, the frictional force does no work.

Change in kinetic energy = work done by the external force

$$\frac{1}{2}mv^2+\frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2=F\cos\theta\cdot x\ \left[v=R\omega\right]$$

$\theta=30°$

$$\therefore\ x=\frac{\frac{3}{4}mv^2}{(F\cos\theta)}=\frac{\frac{3}{4}\times600\times2^2}{500\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)}=2.4\ \text{m}$$

SECTION II

30. (B), (C), (D)

Let x_{01} and x_{02} be the deformation of the springs in equilibrium position. When the block has moved to the right by an amount x_1, other the stretch in spring one is increased by 2x and the stretch in other spring is decreased by 3x.

i.e., In equilibrium, $3\,k_2 \cdot x_{02} - 2\,k_1 \cdot x_{01} = 0$ ... (1)

When the block is displaced by a distance x, the change in elastic potential energy

$$\Delta U = \frac{1}{2}k_1\left(x_{01} + 2x\right)^2 - \frac{1}{2}k_1 x_{01}^2 + \frac{1}{2}k_2\left(x_{02} - 3x\right)^2 - \frac{1}{2}k_2 x_{02}^2$$

$$= \frac{1}{2}\left(4\,k_1 x^2 + 4\,k_1 x\,x_{01} + 9\,k_2 x^2 - 6\,k_2\,x\,x_{02}\right) \quad \text{... (2)}$$

From equation (1) and (2)

$$\Delta U = \frac{1}{2}\left(4\,k_1 + 9\,k_2\right)x^2$$

Change in elastic potential energy = change in kinetic energy

$$\frac{1}{2}mA^2\omega^2 = \frac{1}{2}\left(4k_1 + 9k_2\right)A^2$$

$$\omega = \sqrt{\frac{(4 \times 500 + 9 \times 400)}{8}} = \sqrt{700}\ \text{rad/s}$$

The differential equation of motion is

$$\frac{d^2x}{dt^2} + 700\,x = 0$$

Time period, $T = \frac{2\pi}{\sqrt{700}}$ s

$\omega A = v = 0.8$ m/s

$$A = \frac{0.8}{\sqrt{700}} = 3 \times 10^{-2}\ \text{m}$$

31. (A), (B), (C)

By the law of conservation of linear momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$2 \times 3 + 1(-1) = 2 v_A + v_B$$

By Newton's experimental law

$$0.5 = \frac{-(v_B - v_A)}{(-1) - (3)}$$

$\Rightarrow v_A = 1$ m/s $\qquad v_B = 3$ m/s

Initial kinetic energy $K.E._i = \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2$

$$= \frac{1}{2} \times 2 \times 3^2 + \frac{1}{2} \times 1 \times 1^2$$

$$= 9.5 \text{ J}$$

Final kinetic energy $K.E._f = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$$= \frac{1}{2} \times 2 \times 1^2 + \frac{1}{2} \times 1 \times 3^2$$

$$= 5.5 \text{ J}$$

Percentage decrease in kinetic energy $= \left(\frac{9.5 - 5.5}{9.5}\right) \times 100$

$$= 42\%$$

32. (B), (D)

Let ω_0 be the angular velocity of the earth about its axis

$$\omega_0 = \frac{2\pi}{24} \text{ rad/hr}$$

Let ω be the angular velocity of the satellite

$$\omega = \frac{2\pi}{1.5} \text{ rad/hr}$$

For a satellite rotating from west to east, the relative angular velocity

$$\omega_1 = \omega - \omega_0 = \frac{2\pi}{1.5} - \frac{2\pi}{24}$$

Time period of rotation relative to the earth

$$T_1 = \frac{2\pi}{\omega_1} = \frac{24 \times 1.5}{(24 - 1.5)} = 1.6 \text{ hr}$$

For a satellite rotating from east to west, the relative angular velocity $\omega_2 = \omega + \omega_0$

Time period of rotation $T_2 = \frac{2\pi}{\omega_2} = \frac{24 \times 1.5}{24 + 1.5} = \frac{24}{17}$ hr.

33. (B), (C), (D)

SECTION III

34. (C)

35. (C)

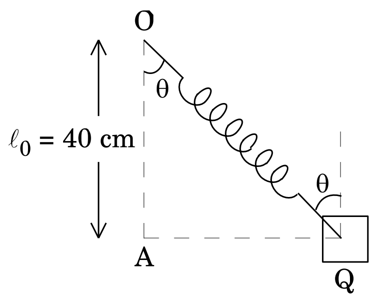
36. (D)

37. (A)

SECTION IV

38. (B) k = 40 N/m

Let A be at Q when it breaks off the plane



Length of the stretched spring = OQ = $\frac{\ell_0}{\cos\theta}$

Increase in the length of spring

$$= \frac{\ell_0}{\cos\theta} - \ell_0$$

Tension in the spring = $k\ell_0 \left(\frac{1}{\cos\theta} - 1\right)$

Vertical component of tension = $k\ell_0 \left(\frac{1}{\cos\theta} - 1\right)\cos\theta$

$$= k\,\ell_0 (1 - \cos\theta)$$

When this is just greater than mg, the block A breaks off the plane

i.e., $k\ell_0 (1 - \cos\theta) = mg$

$$\cos\theta = 1 - \frac{mg}{k\ell_0}$$

$$= \left(1 - \frac{0.32 \times 10}{40 \times 0.4}\right) = 0.8$$

Tension in the spring = $k\ell_0 \left(\frac{1}{\cos\,\theta} - 1\right)$

$= 40 \times 0.4 \left(\frac{1}{0.8} - 1\right)$

= 4 N

39. (A) Gain in elastic potential energy of spring

$U = \frac{1}{2} kx^2$

when 'x' is the extension in the spring

$x_0 = \ell_0 \left(\frac{1}{\cos\,\theta} - 1\right) = 0.4 \left(\frac{1}{4}\right) = 0.1 \text{ m}$

$U = \frac{1}{2} \times 40 \times 10^{-2} = 0.2 \text{ J}$

40. (D) Distance moved by the block 'A' = AQ

$= \ell_0 \tan\theta$

$= 0.4 \times \frac{3}{4} = 0.3$ m

Loss in gravitational potential energy = mgh

= 0.32 × 10 × 0.3 = 0.96 J

Gain in kinetic energy = $2\left(\frac{1}{2} mv^2\right) = (0.96 - 0.2)$ J

$v = \sqrt{\frac{0.76}{0.32}} = \sqrt{\frac{19}{8}}$ m/s

41. (C) Acceleration of rocket $\frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt}$

$$\int_0^v dv = -\int_{m_0}^{m} v_{ex} \frac{dm}{m}$$

$$v = -v_{ex} \ell n \frac{m}{m_0}$$

m_0 = 12000 kg

∴ m = (12000 – 9000) = 3000 kg

$v = -v_{ex} \ell n \frac{3000}{12000} = v_{ex} \ell n\ 4 = 2 v_{ex} \ell n\ 2 = 1.386\, v_{ex}$

42. (B) When the fuel in the first stage is exhausted, speed of the first stage

$$v_1 = v_{ex} \cdot \frac{\ell n(m_0)}{m_1} \text{ where } m_1 = (12000 - 7500)$$

$$= v_{ex}\, \ell n \left(\frac{12000}{4500}\right) = v_{ex}\left(\ell n\left(\frac{8}{3}\right)\right)$$

At this instant, the first stage separates from the second stage. Speed attained becomes the initial speed of second stage.

Final speed of second stage

$$v_2 = v_1 + v_{ex}\, \ell n \left(\frac{2000}{500}\right)$$

$$= v_{ex}\, \ell n \left(\frac{8}{3}\right) + v_{ex}\,(\ell n(4)) = v_{ex}\,[3\,\ell n\, 2 - \ell n\, 3 + 2\,\ell n\, 2]$$

$$= v_{ex}\,[3.466 - 1.099]$$

$$= 2.37\, v_{ex}$$

$$\simeq 2.4\, v_{ex}$$

43. (C) Velocity required $v_{req} = 7.2$ km/s

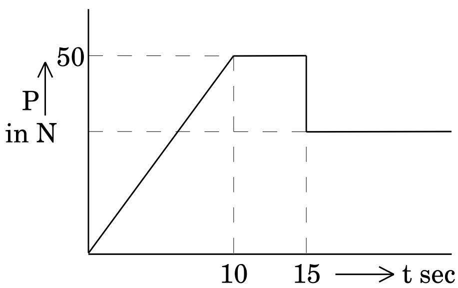
$7.2 \text{ km/s} \simeq v_{ex}\,(2.4)$

$\Rightarrow v_{ex} = 3$ km/s

44. (D) Normal reaction N = mg

$= 10 \times 10 = 100$ N

Limiting frictional force $f_r = \mu_s\, mg = 40$ N



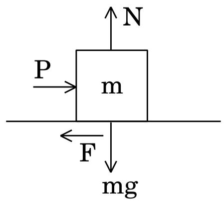
The box begins to slide when $P = f_r$

$$\frac{50}{10}\, t = 40 \Rightarrow t = 8 \text{ s}$$

45. (C) Linear impulse due to the force P acting on the box

from t = 0 to t = 10 s is $\int_0^{10} P\ dt = \frac{1}{2} \times 50 \times 10 = 250$ Ns

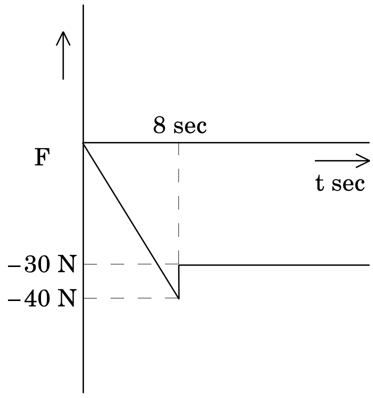
After the box starts to slide the frictional force drops to 0.3 × 100 = 30 N until t_0 when the box again stops moving.



The linear impulse due to the frictional force acting on the box from t = 0 to t = 10 s is

$$\int_0^{10} F\ dt = \frac{1}{2} \times (-40)\,8 + (-30)\,2$$

$$= -220 \text{ Ns}$$



By linear impulse momentum equation

$$\int_0^{10} P\ dt + \int_0^{10} F\ dt = mv_{10}$$

$250 - 220 = 10\ v_{10}$

$\Rightarrow v_{10} = 3$ m/s

46. (D) Velocity at t = 15 s is v_{15}

$$\int_{10}^{15} P\ dt + \int_{10}^{15} F\ dt = mv_{15} - mv_{10}$$

$50\ (15 - 10) - 30\ (15 - 10) = 10\ v_{15} - 10 \times 3$

$\Rightarrow v_{15} = 13$ m/s

Between t = 15 s and t = t_f, the linear impulse momentum equation gives $v_{t_f} = 0$

$$\int_{15}^{t_f} P\ dt + \int_{15}^{t_f} F\ dt = mv_{t_f} - mv_{15}$$

$(25 - 30)\ (t_f - 15) = -10 \times 13$

$t_f = 41$ s

PART C : CHEMISTRY

SECTION I

47. (A) For the reaction $X \longrightarrow Y$

	X	Y
Initial concentration (mol/L)	a	0
After some time "t"	(a – x)	x

In the graph at the point of intersection,

$(a - x) = x$ or $a = 2x$ or $x = \frac{a}{2}$.

Hence 50% of the reaction has occured. Hence time is $t_{50\%}$.

48. (C) $N_2O_4 \rightleftharpoons 2NO_2$

	N_2O_4	$2NO_2$
Initial concentration (mol/L)	1	0
At equilibrium	$(1 - \alpha)$	2α

Total moles before dissociation = 1

Total moles after dissociation = $(1 - \alpha + 2\alpha) = (1 + \alpha)$

Vapour density before dissociation = D = 46

Vapour density after dissociation = d = 24.5

$1 \times D = (1 + \alpha)\, d = d + \alpha d$

$$\alpha = \frac{D - d}{d} = \frac{46 - 24.5}{24.5} = \frac{21.5}{24.5}$$

Percentage of dissociation = $\frac{21.5}{24.5} \times 100 = 87.76 = 88$

49. (C) $2HI \rightleftharpoons H_2 + I_2$

	2HI	H_2	I_2
Initial concentration	100	–	–
At equilibrium	100 – 22 = 78	11	11

$$K = \frac{[H_2][I_2]}{[HI]^2} = \frac{11 \times 11}{78 \times 78} = 0.0199$$

50. (B) $NH_4HS_{(s)} \rightleftharpoons NH_{3(g)} + H_2S_{(g)}$

The equilibrium constant, $K_p = P_{NH_3} \cdot P_{H_2S}$

Pressure of equilibrium mixture = 2 atm.

$$P = P_{NH_3} + P_{H_2S}$$

Since $P_{NH_3} = P_{H_2S} = 1$ atm

So that P = 1 + 1 = 2 atm

$K_p = P_{NH_3} \cdot P_{H_2S}$

$K_p = 1 \times 1\ atm^2 = 1\ atm^2$

51. (D) The Henderson equation for the buffer solution is

$$pH = pK_a + \log \frac{[salt]}{[acid]}$$

$$pH = pK_a + \log \frac{[HCO_3^-]}{[H_2CO_3]}$$

$$7.4 = 6.376 + \log \frac{[HCO_3^-]}{[H_2CO_3]}$$

$$\log \frac{[HCO_3^-]}{[H_2CO_3]} = 7.4 - 6.376 = 1.024$$

Taking antilogarithm,

$$\frac{[HCO_3^-]}{[H_2CO_3]} = 10^{1.024}$$

52. (C) $\Delta T_f = i \times K_f \times$ molality

$$i = \frac{\Delta T_f}{K_f \times \text{molality}}$$

$$= \frac{0.279}{1.86 \times 0.1} = 1.5$$

Since for HA $\rightleftharpoons$ $H^+ + A^-$

$$\alpha = \frac{i-1}{n-1} = \frac{i-1}{2-1} = \frac{1.5-1}{1} = 0.5$$

$[H^+] = \alpha C = 0.5 \times 0.1 = 0.05$ M

$pH = -\log [H^+]$

$$= \log \frac{1}{[H^+]}$$

$$= \log \frac{1}{0.05} = \log 20 = 1.3010$$

SECTION II

53. (A), (B)

54. (A), (C)

Rate = $k[A]^2 [B]$

$$-\frac{d[A]}{dt} = k[A]^2 [B]$$

$$-\frac{1}{3}\frac{d[B]}{dt} = k[A]^2 [B]$$

55. (A), (B), (D)

Copper is less reactive than Mg and Fe.

56. (A), (C), (D)

The first element in the group has high electronegativity.

SECTION III

57. (D) Equilibrium constant at constant temperature is not affected by changing the concentrations of reactant or product.

58. (B) 10^{-7} M of H^+ produced by water is not to be taken as negligible in comparison with 10^{-8} M of H^+ produced by HCl, which is a strong electrolyte in aqueous medium.

59. (C) Catalyst does not affect the ΔG of the reaction but decreases the E_a of the reaction.

60. (C) Helium is inert but beryllium is reactive as it is an alkaline earth metal.

SECTION IV

61. (D) Elements belonging to group 17 are halogens. The melting point increases from fluorine to iodine. But in groups 1, 2 and 13, the melting points decrease downwards.

62. (B) The expected electronic configuration for Pd is $4d^8 5s^2$. Its actual configuration is $4d^{10} 5s^0$. Hence, it belongs to 5th period, group 10.

63. (B) Among the isoelectronic ions, the cations belong to the next period.

64. (C) $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$c(1-\alpha) \qquad \alpha c \qquad \alpha c$

where c is the molarity of CH_3COOH solution

$$K_a = \frac{\alpha^2 c}{(1-\alpha)} \approx \alpha^2 c$$

$$1.8 \times 10^{-5} = \alpha^2 \times 0.09$$

$$\alpha^2 = \frac{1.8 \times 10^{-5}}{0.09} = \frac{1.8 \times 10^{-5}}{0.9 \times 10^{-1}} = 2 \times 10^{-4}$$

$$\alpha = \sqrt{2 \times 10^{-4}} = 1.414 \times 10^{-2}$$

65. (A) The $[H^+]$ is contributed by acetic acid as well as by hydrochloric acid. But contribution from HCl is greater and from acetic acid is negligible.

$$[H^+] = 0.01 + 0.09 \times 1.414 \times 10^{-2} \approx 0.01\ M$$

$$K_a = \frac{[CH_3COO^-][H^+]}{[CH_3COOH]} = \frac{\alpha c \times 0.01}{0.09}$$

$$1.8 \times 10^{-5} = \frac{\alpha \times 0.09 \times 0.01}{0.09}$$

$$\alpha = \frac{1.8 \times 10^{-5} \times 9 \times 10^{-2}}{9 \times 10^{-2} \times 1 \times 10^{-2}} = 1.8 \times 10^{-3}$$

The α value is suppressed in the presence of 0.01 M HCl.

66. (D) $\left[CH_3COO^-\right]$ is decided by sodium acetate only as it is a strong electrolyte and ionisation of acetic acid is suppressed.

$$K_a = \frac{\left[CH_3COO^-\right]\left[H^+\right]}{\left[CH_3COOH\right]}$$

$$1.8 \times 10^{-5} = \frac{0.01 \times 0.09\ \alpha}{0.09} = 0.01\,\alpha$$

$$\alpha = \frac{1.8 \times 10^{-5}}{0.01} = \frac{1.8 \times 10^{-5}}{1 \times 10^{-2}} = 1.8 \times 10^{-3}$$

67. (B) $k_{for} = 2 \times 10^7\ L^2\ mol^{-2}\ s^{-1}$ and $k_{rev} = 1 \times 10^{-2}\ s^{-1}$

$$\frac{k_{for}}{k_{rev}} = \frac{2 \times 10^7}{1 \times 10^{-2}} = 2 \times 10^9\ L^2\ mol^{-2}$$

68. (A) $2A + B \longrightarrow$ Product

The rate law for the reaction is

$$-\frac{d[A]}{dt} = k[A]$$

Hence, it is the first order reaction. The rate equation is

$$k = \frac{1}{t}\ \ell n\ \frac{C_0}{C}$$

When $t = \frac{1}{k}$, $k = \frac{1}{t}$; hence, $\frac{1}{t} = \frac{1}{t}\ \ell n\ \frac{C_0}{C}$ or $\frac{C_0}{C} = e$

or $C = \frac{C_0}{e}$

69. (B) $Zn + 2H^+ \longrightarrow Zn^{2+} + H_2$

Half-life is independent of concentration of Zn.

Hence, the order is one with respect to zinc.

$T_{50\%}$ = 10 minutes when pH = 2 or $[H^+] = 10^{-2}$ M

$T_{50\%}$ = 100 minutes when pH = 3 or $[H^+] = 10^{-3}$ M

i.e., $T_{50\%}$ is inversely proportional to $[H^+]$.

Hence, the order is two with respect to $[H^+]$.

Hence, rate law is, Rate $= k\ [Zn]\ [H^+]^2$.